**Beta & Gamma Functions**

**Beta Function or First Eulerian Integral:** A function of the form,



is called Beta function or first Eulerian integral and it is denoted by, .

.

**Gamma Function or Second Eulerian Integral:** A function of the form,



is called Gamma function or second Eulerian integral and it is denoted by, .

.

**Properties of Beta and Gamma functions:** The properties are given below:

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. .
9. **.**

**Theorem-01:** Prove that .

**Proof:** We know,the beta function is



Let 





From (1) we get,





  **(Proved)**

**Theorem-02:** Prove that .

**Proof:** We know,the beta function is



If  then,

































  **(Proved)**

**Theorem-03:** Prove that i).  ; ii). .

**Proof:** We know,the Gamma function is



If  then, from (1) we get,













  **(Proved)**

Again, replacing  by in (1) we get,



 [Integrating by parts]





 **(Proved)**

**Theorem-04:** Establish the relation between Gamma and Beta function.

**Or,** Prove that .

***Proof:*** From the definition of Gamma function we can write



Assume  .

Limit: when , then and when , then .

From above relation we have







Again,



Multiplying (i) and (ii) we get









 



 

 **(Proved)**

**Theorem-05:** Prove that ****

**Proof:** We know that

Let .

Limit: .

Now,









Assume .

Now from above equation we get



Using the relation between beta and gamma function**,** we have



 **(Proved)**

**Problem-01:** Evaluate  **Exer.-01:** 

**Solution:** Let,  **Ans:** 

 **Exer.-02:** 

 **Ans:** 

 **Exer.-03:** 

 **Ans:** 







 **(Ans.)**

**Problem-02:** Evaluate 

**Solution: Solution:** Let, 















 **(Ans.)**

**Problem-03:** Evaluate  **Exer.-04:** 

**Solution:** Let,  **Ans:** 

 **Exer.-05:** 

 **Ans:** 

 **Exer.-06:** 

 **Ans:** 

 **(Ans.)**

**Problem-04:** Evaluate  **Exer.-04:** 

**Solution:** Let,  **Ans:** 

 **Exer.-05:** 

 **Ans:** 

















 **(Ans.)**

**Problem-05:** Evaluate  **Exer.-06:** 

**Solution:** Let,  **Ans:** 

















 **(Ans.)**

**Problem-06:** Evaluate  **Exer.-07:** 

**Solution:** Let,  **Ans:** 

 **Exer.-08:** 

 **Ans:** 























 **(Ans.)**

**Problem-08:** Evaluate  **Exer.-09:** 

**Solution:** Let,  **Ans:** 

Put  **Exer.-10:** 

 **Ans:** 

Now,  **Exer.-11:** 

 **Ans:** 













  **(Ans.)**

**Problem-09:** Evaluate  **Exer.-12:** 

**Solution:** Let,  **Ans:** 

Put 



Now, 















  **(Ans.)**

**Problem-10:** Evaluate 

**Solution:** Let, 

Put 



Now, 



















  **(Ans.)**

**Problem-11:** Evaluate  **Exer.-13:** 

**Solution:** Let,  **Ans:** 











 **(Ans.)**

**Problem-12:** Show that  **Exer.-14: Show that** 

**Solution:** Let,  **Exer.-15: Show that** 

Put, 





Now, 









 **(Showed.)**

**Problem-13:** Show that  **Exer.-16: Show that** 

**Solution:** Let, 











 **(Showed)**

**Problem-14:** Show that  **Exer.-17: Show that** 

**Solution:** Let, 











 **(Showed.)**